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Hierarchical Supersymmetry Breaking in Superstring Derived Standard–like Models

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ABSTRACT

We examine the problem of supersymmetry breaking in realistic superstring standard–like models which are constructed in the free fermionic formulation. We impose a supersymmetric vacuum at the Planck scale by requiring vanishing F and D constraints at the cubic level of the superpotential. We then study possible scenarios for supersymmetry breaking by examining the role of nonrenormalizable terms and hidden sector gaugino and matter condensates. We argue that in some scenarios hierarchical supersymmetry breaking in the observable sector is possible.

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1. Introduction

Experiments at present day accelerators indicate that the Standard Model correctly accounts for the observed particle spectrum and interactions. Due to its renormalizability, the Standard Model may be the correct theory up to a very large energy scale. Indeed, for the last two decades this possibility has been exploited in the development of Grand Unified Theories and superstring theories [1]. However, in this context, two fundamental problems are raised: the stabilization of the scalar sector and the derivation of the low energy scale from the fundamental high energy scale. Supersymmetry and supergravity [2] have been advocated as possible solutions to both of these problems. However, as supersymmetry is not a symmetry of the observed particle spectrum and interactions, it is at best a broken symmetry. The problem of supersymmetry breaking received much attention both in the context of supergravity and superstring theories [3]. In supergravity, one has to impose the existence of a hidden sector. Strong hidden sector dynamics are responsible for breaking supersymmetry in the hidden sector, which is transmitted to the observable sector by gravitational interactions. In the context of superstring theories, the hidden sector arises naturally from the consistency of the theory.

On the other hand, superstring theory is the only known candidate for a consistent unified theory of quantum gravity and the gauge interactions. As such it provides a unique opportunity to study how the parameters of the Standard Model may arise from a fundamental Planck scale theory. Luckily, to produce a fermion spectrum string theory must accommodate $N = 1$ space–time supersymmetry, at least at the Planck scale. Several attempts have been made to construct semi–realistic superstring models [4–8]. Of those, the most notable are the realistic models [4,6,7,8] in the free fermionic formulation [9]. Due to proton decay constraints, the most compelling choice is to derive the Standard Model directly from the superstring [8,10]. The superstring derived standard–like models of Refs. [6,7,8] exemplify how the Standard Model may be derived directly from superstring theory. In Refs. [8,10,11,12,13,14,15] some of the fundamental problems in particle physics were addressed in the context of these models. Among them, proton de-

cay [8,10], gauge coupling unification [11], generation mass hierarchy [12], neutrino masses [13], texture of fermion mass matrices [10,12,14], quark flavor mixing [14], and axions [15].

In this paper, we study the problem of supersymmetry breaking in realistic superstring derived standard–like models. These models correspond to $Z_2 \times Z_2$ orbifold models with nontrivial background fields and with three additional Wilson lines [16]. Three additional Wilson lines are needed to reduce the number of generations to three, one from each one of the twisted sectors of the corresponding $Z_2 \times Z_2$ orbifold models. The standard–like models contain a hidden gauge group which is some subgroup of E_8, typically $SU(5) \times SU(3) \times U(1)^2$, with matter spectrum in vector–like representations. An important property of the standard–like models is the existence of an "anomalous" $U(1)$ symmetry, which is instrumental in determining the parameters of the low energy effective theory and in lifting the degeneracy among string vacua.

The problem of supersymmetry breaking is divided into two parts. The first is the determination of the compactification parameters, *i.e.* the dilaton VEV, the moduli VEVs and the singlet VEVs which cancel the anomalous $U(1)$ D–term equation. The second is the hierarchical breaking of supersymmetry in the observable sector, given a supersymmetric vacuum at the Planck scale. In this paper, we do not address the first part of the problem. We assume that these VEVs are fixed by some unknown, possibly nonperturbative, string mechanism. Thus, given a set of compactification parameters with a supersymmetric vacuum at the Planck scale and at the cubic level of the superpotential, we examine a possible scenario for supersymmetry breaking at hierarchically low energies. In our approach, we impose a supersymmetric vacuum at the Planck scale by requiring vanishing F and D flatness constraints at the cubic level of the superpotential. The $SO(10)$ singlet VEVs that we impose are motivated by requiring quark masses and mixing of the correct order of magnitude. By imposing vanishing F and D flat directions at the cubic level of the superpotential we guarantee that there are no supersymmetry breaking terms of the order of M_{Pl}. We then study the effects of strong hidden sector dy-

namics and nonrenormalizable terms on the effective superpotential. We find that if only observable sector states obtain nonvanishing VEVs in the cancelation of the "anomalous" $U(1)$ D–term equation, supersymmetry is unbroken to all orders of nonrenormalizable terms. On the other hand, if some hidden sector states obtain nonvanishing VEVs, then supersymmetry is broken due to nonrenormalizable terms and hidden sector matter condensates. When hidden sector matter condensation occurs, the cubic level F constraints are modified to a set of equations that cannot be satisfied simultaneously. Consequently, nonvanishing F–terms are generated which break supersymmetry at a hierarchically small scale in the observable sector.

Our paper is organized as follows. In section 2, we review the superstring standard–like models and the properties that are important for the discussion that follows. In section 3, we discuss the cubic level superpotential and phenomenological constraints on the Standard Model singlet VEVs. In section 4, we examine the effects of nonrenormalizable terms and the strong hidden sector dynamics. Section 5 contains a discussion of several problems related to supersymmetry breaking and our conclusions.

2. The superstring standard–like models

The superstring standard–like models [7,6,8] are constructed in the four dimensional free fermionic formulation [9]. To study the problem of supersymmetry breaking we focus on the model that was presented in Ref. [6]. The standard–like models are generated by sets of eight basis vectors, $\{\mathbf{1}, S, b_1, b_2, b_3, \alpha, \beta, \gamma\}$. The set $\{\mathbf{1}, S, b_1, b_2, b_3, 2\gamma\}$ is common to all the realistic models in the free fermionic formulation. The set $\{\mathbf{1}, S, \mathbf{1} + b_1 + b_2 + b_3, 2\gamma\}$ generates a toroidal compactified model with $N = 4$ space–time supersymmetry and $SO(12) \times SO(16) \times SO(16)$ gauge symmetry. The vectors b_1 and b_2 correspond to moding out the six dimensional torus by a $Z_2 \times Z_2$ discrete symmetry with standard embedding, [12,16]. The vectors α, β, γ differ between models and correspond to different choices of Wilson lines in the orbifold language. The various choices of vectors α, β, γ and

of the phases $c\begin{pmatrix}\alpha,\beta,\gamma\\ \mathbf{1},S,b_i\end{pmatrix}$ fix the physical spectrum and determine the low energy effective theory of the superstring standard–like models.

The full massless spectrum together with the quantum numbers were given in Ref. [6]. The gauge group after all GSO projections have been applied is* $\{SU(3)\times SU(2)\times U(1)_C\times U(1)_L\times U(1)^6\}_O\times\{SU(5)_H\times SU(3)_H\times U(1)^2\}_H$, where the first curly brackets correspond to the observable gauge group that arises from the first $SO(16)$ times $SO(12)$. The second curly brackets correspond to the hidden gauge group that arises from the second $SO(16)$. Here by hidden gauge group we mean that the states that are identified with the three generations do not transform under the hidden gauge group. The weak hypercharge is uniquely given by $U(1)_Y={1\over3}U(1)_C+{1\over2}U(1)_L$. The orthogonal combination is given by $U(1)_{Z'}=U(1)_C-U(1)_L$. The supersymmetry generator is the basis vector S and the superpartners of the states from a given sector α are obtained from the sector $S+\alpha$. The sectors b_1, b_2 and b_3 correspond to the three twisted sectors of the orbifold model and produce three 16 of $SO(10)$ decomposed under $SU(3)\times SU(2)\times U(1)_C\times U(1)_L$ with charges under the horizontal symmetries. For every generation, G_j there are two right–moving, $U(1)_{r_j}$ and $U(1)_{r_{j+3}}$, symmetries. For every right–moving $U(1)$ gauged symmetry, there is a corresponding left–moving global $U(1)$ symmetry, $U(1)_{\ell_j}$ and $U(1)_{\ell_{j+3}}$. Each sector b_1, b_2 and b_3 has two Ising model operators, (σ_4,σ_5), (σ_2,σ_6) and (σ_1,σ_3), respectively, obtained by pairing a left–handed real fermion with a right–handed real fermion.

The Neveu–Schwarz (NS) sector corresponds to the untwisted sector of the orbifold model and produces in addition to the gravity and gauge multiplets three pairs of electroweak scalar doublets $\{h_1,h_2,h_3,\bar h_1,\bar h_2,\bar h_3\}$, three pairs of $SO(10)$ singlets with $U(1)$ charges, $\{\Phi_{12},\Phi_{23},\Phi_{13},\bar\Phi_{12},\bar\Phi_{23},\bar\Phi_{13}\}$, and three scalars that are singlets of the entire four dimensional gauge group, ξ_1,ξ_2,ξ_3.

The sector $S+b_1+b_2+\alpha+\beta$ ($\alpha\beta$ sector) also produces states that transform

$*$ $U(1)_C={1\over2}U(1)_{B-L}$, $U(1)_L={1\over2}U(1)_{T_{3_R}}$.

only under the observable gauge group. In addition to one pair of electroweak doublets, $h_{45}, \bar{h}_{45}$, and one pair of color triplets, there are seven pairs of $SO(10)$ singlets with horizontal $U(1)$ charges, $\{\Phi_{45}, \bar{\Phi}_{45}, \Phi^{\pm}_{1,2,3}, \bar{\Phi}^{\pm}_{1,2,3}\}$.

In addition to the states from these sectors, which transform solely under the observable gauge group, the sectors $b_j + 2\gamma$ produce states which are $SO(10)$ singlets and transform as the 16 vector representation of the hidden $SO(16)$, decomposed under $SU(5) \times SU(3) \times U(1)^2$, $\{T_{1,2,3}, \bar{T}_{1,2,3}, V_{1,2,3}, \bar{V}_{1,2,3}\}$. The T_i ($\bar{T}_i$) are 5 ($\bar{5}$) and the V_i ($\bar{V}_i$) are 3 ($\bar{3}$) of the hidden $SU(5)$ and $SU(3)$ gauge groups respectively (in order not to cause any confusion between the hidden sector states T_i and the moduli we will call the moduli $\hat{T}_i$ throughout the paper). These states arise due to the $Z_2 \times Z_2$ twist of $SO(12) \times SO(16) \times SO(16)$ rather than of $SO(12) \times E_8 \times E_8$, in which they are replaced by $10 + 1$ of $SO(10)$ and thus complete the 16 of $SO(10)$ to 27 of E_8. These states are charged under the horizontal $U(1)_{r_j}$ symmetries and play an important role in generating quark flavor mixing.

The vectors with some combination of $(b_1, b_2, b_3, \alpha, \beta) \pm \gamma + (I)$ produce additional states in vector–like representations. Most of those are Standard Model singlets but carry nonvanishing $U(1)_{Z'}$ charge, where $U(1)_{Z'}$ is the $U(1)$ inside $SO(10)$ that is orthogonal to the electroweak hypercharge. The sectors $b_{1,2} + b_3 + \alpha \pm \gamma + (I)$ also produce a pair of $SU(3)_C$ triplets and a pair of electroweak doublets.

The cubic level superpotential and higher order nonrenormalizable terms in the superpotential are obtained by calculating correlators between vertex operators, $A_N \sim \langle V_1^f V_2^f V_3^b \cdots V_N^b \rangle$, where V_i^f (V_i^b) are the fermionic (scalar) components of the vertex operators. The nonvanishing terms must be invariant under all the symmetries of the string models and must satisfy all the string selection rules [19]. To obtain the correct ghost charge, $(N_b - 1)$ of the bosonic vertex operators have to be picture changed from the -1 ghost picture to the 0 ghost picture. The invariance under the six global left–moving $U(1)$ symmetries and the Ising model correlators must be checked after all picture changing operations have been performed. The invariant terms are extracted by using a simple FORTRAN code.

The cubic level superpotential is given by [6],

$$
\begin{aligned}
W = \{ & (u^c_{L_1} Q_1 \bar{h}_1 + N^c_{L_1} L_1 \bar{h}_1 + u^c_{L_2} Q_2 \bar{h}_2 + N^c_{L_2} L_2 \bar{h}_2 + u^c_{L_3} Q_3 \bar{h}_3 + N^c_{L_3} L_3 \bar{h}_3) \\
& + h_1 \bar{h}_2 \bar{\Phi}_{12} + h_1 \bar{h}_3 \bar{\Phi}_{13} + h_2 \bar{h}_3 \bar{\Phi}_{23} + \bar{h}_1 h_2 \Phi_{12} + \bar{h}_1 h_3 \Phi_{13} + \bar{h}_2 h_3 \Phi_{23} \\
& + \Phi_{12}(\Phi_{23} \bar{\Phi}_{13} + \Phi^-_1 \Phi^+_1 + \Phi^-_2 \Phi^+_2 + \Phi^-_3 \Phi^+_3) \\
& + \bar{\Phi}_{12}(\bar{\Phi}_{23} \Phi_{13} + \bar{\Phi}^+_1 \bar{\Phi}^-_1 + \bar{\Phi}^+_2 \bar{\Phi}^-_2 + \bar{\Phi}^+_3 \bar{\Phi}^-_3) \\
& + \frac{1}{2} \xi_3 (\Phi_{45} \bar{\Phi}_{45} + \Phi^+_1 \bar{\Phi}^+_1 + \Phi^-_1 \bar{\Phi}^-_1 + \Phi^+_2 \bar{\Phi}^+_2 + \Phi^-_2 \bar{\Phi}^-_2 + \Phi^+_3 \bar{\Phi}^+_3 + \Phi^-_3 \bar{\Phi}^-_3 \\
& + h_{45} \bar{h}_{45} + D_{45} \bar{D}_{45}) + h_3 \bar{h}_{45} \Phi_{45} + \bar{h}_3 h_{45} \bar{\Phi}_{45} \} \\
& + \{ \frac{1}{2} [\xi_1 (H_{19} H_{20} + H_{21} H_{22} + H_{23} H_{24} + H_{25} H_{26}) \\
& + \xi_2 (H_{13} H_{14} + H_{15} H_{16} + H_{17} H_{18})] + \bar{\Phi}_{23} H_{24} H_{25} + \Phi_{23} H_{23} H_{26} \\
& + h_2 H_{16} H_{17} + \bar{h}_2 H_{15} H_{18} + e^c_{L_1} H_{10} H_{27} + e^c_{L_2} H_8 H_{29} + H_{27} (V_1 H_9 \\
& + V_2 H_{11}) + V_6 H_5 H_{29} + \bar{\Phi}_{45} H_{17} H_{24} + D_{45} H_{18} H_{21} + h_{45} H_{16} H_{25} \}
\end{aligned} \qquad (1)
$$

where a common normalization constant $\sqrt{2}g$ is assumed. From Eq. (1) it is seen that only $+\frac{2}{3}$ charged quarks obtain a cubic level mass term. This result arises due to the assignment of boundary conditions in the vector γ [8]. Mass terms for $-\frac{1}{3}$ and for charged leptons must be obtained from nonrenormalizable terms.

An important property of the superstring standard–like models is the absence of gauge and gravitational anomalies apart from a single "anomalous $U(1)$" symmetry. This "anomalous" $U(1)_A$ generates a Fayet–Iliopoulos term that breaks supersymmetry at the Planck scale [17]. Supersymmetry is restored and $U(1)_A$ is broken by giving VEVs to a set of Standard Model singlets in the massless string spectrum along the flat F and D directions [18]. The $SO(10)$ singlet fields in the nonrenormalizable terms obtain nonvanishing VEVs by the application of the DSW mechanism. Thus the order N nonrenormalizable terms, of the form $cffh(\Phi/M)^{N-3}$, become effective trilinear terms, where f, h, Φ denote fermions, scalar doublets and scalar $SO(10)$ singlets, respectively, and $M = M_{Pl}/(2\sqrt{8\pi}) \approx 10^{18} GeV$. The effective Yukawa couplings are given by $\lambda = c(\langle\Phi\rangle/M)^{N-3}$, where the calculable coefficients c are of order one [19], and are suppressed relative to the Yukawa couplings that

are obtained at lower orders (since $\langle\Phi\rangle \sim M/10$ generically). In this manner quark mass terms, as well as quark mixing terms, can be obtained. Realistic quark masses and mixing can be obtained for a suitable choice of scalar VEVs. Requiring quark mixing angles of the correct order of magnitude necessitates that we give nonvanishing VEVs to some states from the sectors $b_j + 2\gamma$ in the application of the DSW mechanism.

The form of the effective supergravity theory in the free fermionic model was analyzed in Ref. [20,21]. The Kähler potential takes the form

$$K = -\ln(S + S^\dagger) - \sum_{a=1}^{3} \ln {\eta_a}^0 + \sum_\alpha 2\ln(1 - \phi_\alpha {\phi_\alpha}^\dagger)\ , \tag{2a}$$

where

$$ {\eta_a}^0 = \frac{1}{\sqrt{2}} \left| 1 + |\hat\eta_a^1|^2 - \sum_{i_a=2}^{n_a} |\hat\eta_a^{i_a}|^2 \right|^{\frac{1}{2}}\ , \tag{2b}$$

$$ {\hat\eta_a^1}^2 = -1 + \sum_{i_a=2}^{n_a} (\hat\eta_a^{i_a})^2, \tag{2c}$$

where S and ϕ_i are the dilaton and chiral matter fields and $(\eta_a^0, \eta_a^1, \eta_a^{i_a})$ correspond to the physical and unphysical components of the moduli fields [20,21]. The value of e^K is determined by the expectation values of S, of the gauge non–singlet $\hat\eta_a^{i_a}$ and of the matter fields. In the free fermionic models, the fields that correspond to the moduli fields in orbifold models, appear in D–terms and possibly have nontrivial superpotential couplings. Their VEVs are subject to the F and D–flatness constraints.

To study the problem of supersymmetry breaking, we have to relate the effective field theory that is obtained in the free fermionic models to the one that is used in supergravity. In most analyses of supersymmetry breaking in superstring inspired field theories, one assumes the Kähler potential at tree level to take the

form:

$$K = -\ln(S + S^\dagger) - 3\ln(\hat{T} + \hat{T}^\dagger - \sum_i |\phi_i|^2), \tag{3}$$

where S and $\hat{T}$ are the dilaton and overall modulus field and the ϕ_i are the gauge nonsinglet chiral superfields. The effective supergravity action is invariant under discrete target space duality transformations [22],

$$\hat{T}_j \to \frac{a_j \hat{T}_j - i b_j}{i c_j \hat{T}_j + d_j}, \tag{4}$$

where $j = 1, 2, 3$ and a_j, b_j, c_j, d_j are integers which satisfy $a_j d_j - b_j c_j = 1$. This symmetry has been used extensively to constrain the superstring inspired effective field theory and was also discussed in S-matrix approach to the effective string field theory [23]. In the free fermionic models, if we assume that the fields that correspond to the moduli are correctly identified, then we may assume a Kähler potential of the form of Eq. (3). The modular group of the free fermionic models is $PSL(2, Z)^3$ and the associated moduli fields are denoted by $\hat{T}_j$ ($j = 1, 2, 3$). The modular weight of each matter field is related to the charges under the left–moving $U(1)_{\ell_{1,2,3}}$ symmetries, where $U(1)' = U(1)_1 + U(1)_2 + U(1)_3$ is the $U(1)$ current in the left–moving $N = 2$ world–sheet supersymmetry. The matter fields then transform as

$$\phi_i \to \phi_i \prod_j (i c_j \hat{T}_j + d_j)^{-w^i_j}, \tag{5}$$

under modular transformations, where $-w^i_j$ are the charges of the scalar component of the ϕ_i superfields under the left–moving $U(1)_{\ell_j}$ ($j = 1, 2, 3$) symmetries. Under target space duality transformation the superpotential has overall weight -3 and the dilaton has weight zero. Modular invariance is manifest at the cubic level of the superpotential. The existence of a nonvanishing term at some order N implies the existence of an infinite number of nonrenormalizable terms. These terms are obtained by tagging, to the order N term, powers of scalar fields in the massless

spectrum that correspond to the moduli fields [24]. Tagging moduli to the order N term corresponds to multiplying the order N term by an appropriate power of the Dedekind η function, with modular weight $1/2$, which renders the theory with the nonrenormalizable terms modular invariant. However, as the corrections from the infinite tower of nonrenormalizable terms are suppressed by powers of $(\langle\phi\rangle/M)^n$, for our purposes we will neglect the modular invariance of the effective theory, asserting that the full theory will only induce small corrections to our results. Thus, we use the following Kähler function for our calculation:

$$K = -\ln(S+S^\dagger) - \sum_{a=1}^{3} \ln {\eta_a}^0 + f(\hat{T},\hat{T}^\dagger)\phi_i\phi_i^\dagger, \tag{6}$$

where $f(\hat{T},\hat{T}^\dagger)$ is a function that may depend on the moduli.

3. Cubic level F and D-constraints

We now proceed to analyze the F and D-flatness constraints in the cubic level superpotential. The set of F and D constraints is given by the following equations:

$$D_A = \sum_k Q_k^A|\chi_k|^2 = \frac{-g^2 e^{\phi_D}}{192\pi^2}\frac{1}{2\alpha'}Tr(Q_A) \tag{7a}$$

$$D'_j = \sum_k {Q'}_k^j|\chi_k|^2 = 0 \quad j = 1\cdots 5 \tag{7b}$$

$$D_j = \sum_k Q_k^j|\chi_k|^2 = 0 \quad j = C, L, 7, 8 \tag{7c}$$

$$W = \frac{\partial W}{\partial \eta_i} = 0 \tag{7d}$$

where χ_k are the fields that get a VEV and Q_k^j is their charge under the $U(1)_j$ symmetry. The set $\{\eta_i\}$ is the set of fields with vanishing VEV. α' is the string tension and $1/(\sqrt{2\alpha'}) = gM_{Pl}/(2\sqrt{8\pi}) = gM \sim 10^{18}\ GeV$. In the model of Ref. [6], $Tr(Q_A) = 180$.

The observable sector F flatness conditions derived from the cubic superpotential are:

$$\bar{\Phi}_{13}\Phi_{12} + H_{23}H_{26} = \Phi_{13}\bar{\Phi}_{12} + H_{24}H_{25} = 0 \tag{8a}$$

$$\Phi_{23}\Phi_{12} = \bar{\Phi}_{23}\bar{\Phi}_{12} = 0 \tag{8b}$$

$$\bar{\Phi}_{23}\Phi_{13} + \bar{\Phi}_i^{+}\bar{\Phi}_i^{-} = 0 \tag{8c}$$

$$\bar{\Phi}_i^{+}\bar{\Phi}_{12} + \Phi_i^{-}\xi_3 = 0 \tag{8d}$$

$$\bar{\Phi}_i^{-}\bar{\Phi}_{12} + \Phi_i^{+}\xi_3 = 0 \tag{8e}$$

$$\Phi_{45}\bar{\Phi}_{45} + \Phi_i^{+}\bar{\Phi}_i^{+} + \Phi_i^{-}\bar{\Phi}_i^{-} = 0 \tag{8f}$$

$$\Phi_{45}\xi_3 + H_{17}H_{24} = 0 \tag{8g}$$

$$\bar{\Phi}_{45}\xi_3 = 0. \tag{8h}$$

For equations $(8c) - (8e)$ the barred equations have to be taken as well. From the $\langle D_A \rangle = 0$ constraint we can show that Φ_{45} must get a nonvanishing VEV. We can always modify the set of D constraints in a way that only the coefficient of $\langle \Phi_{45} \rangle$ is different from zero and consequently Φ_{45} must get a VEV to cancel the "anomalous" $U(1)$ D–term equation. From Eq. (8g) it follows that $\langle \xi_3 \rangle = 0$ because $\langle H_{17} \rangle = 0$, as we will see below. In general, some additional $\Phi_i^{\pm}$ and $\bar{\Phi}_i^{\pm}$ obtain VEVs from the set of D–term constraints. Therefore, from Eqs. (8d-8e) it follows that $\langle \Phi_{12} \rangle = 0$ and $\langle \bar{\Phi}_{12} \rangle = 0$. We conclude that the cubic level F–flatness constraints in the observable sector impose

$$\langle \Phi_{12}, \bar{\Phi}_{12}, \xi_3 \rangle = 0. \tag{9}$$

In the hidden sector, on the other hand, we get the following F constraints from the cubic superpotential:

$$H_{19}H_{20} + H_{23}H_{24} + H_{25}H_{26} = 0 \tag{10a}$$

$$H_{13}H_{14} + H_{17}H_{18} = 0 \tag{10b}$$

$$\frac{1}{2}\xi_1 H_{24} + \Phi_{23} H_{26} = 0 \qquad (10c)$$

$$\frac{1}{2}\xi_1 H_{23} + \bar{\Phi}_{23} H_{25} = 0 \qquad (10d)$$

$$\frac{1}{2}\xi_1 H_{26} + \bar{\Phi}_{23} H_{24} = 0 \qquad (10e)$$

$$\frac{1}{2}\xi_1 H_{25} + \Phi_{23} H_{23} = 0 \qquad (10f)$$

$$\frac{1}{2}\xi_2 H_{13} = \frac{1}{2}\xi_2 H_{14} = \frac{1}{2}\xi_2 H_{17} = \frac{1}{2}\xi_2 H_{18} = 0. \qquad (10g)$$

The requirement of realistic (or nonzero) b, s, μ, τ masses imposes that ξ_1 and ξ_2 must get VEVs [10,12]. Below we will show that $\langle \xi_1, \xi_2 \rangle \neq 0$ also insures the stability of the supersymmetric vacuum. Then, from Eq. (10g) we obtain

$$\langle H_{13} \rangle = \langle H_{14} \rangle = \langle H_{17} \rangle = \langle H_{18} \rangle = 0, \qquad (11)$$

in order to preserve supersymmetry at M_{Pl}. H_{19} and H_{20} are 5 and $\bar{5}$ of $SU(5)$. In the scenario that we study we assume that the hidden $SU(5)$ group is unbroken at M_{Pl}, which imposes $\langle H_{19} H_{20} \rangle = 0$. For the rest, $H_{23}, H_{24}, H_{25}, H_{26}$, from Eqs. (10c–f) we get the following constraints:

a) When $\langle \Phi_{23} \rangle \neq 0$ and $\langle \bar{\Phi}_{23} \rangle \neq 0$, either $\langle H_{23} \rangle = \langle H_{25} \rangle = 0$ or $\langle H_{24} \rangle = \langle H_{26} \rangle = 0$.

b) When one or both of $\Phi_{23}, \bar{\Phi}_{23}$ have vanishing VEVs, $\langle H_{23} \rangle = \langle H_{25} \rangle = \langle H_{24} \rangle = \langle H_{26} \rangle = 0$.

We stress that these supersymmetry constraints on the hidden sector VEVs are obtained by requiring a realistic heavy quark and lepton spectrum (i.e. $\langle \xi_{1,2} \rangle \neq 0$) and a nonsingular hidden matter mass matrix. Otherwise, Eqs. (10a–g) do not lead to useful supersymmetry constraints on the hidden sector VEVs.

Given the assumption that VEVs that break $U(1)_{Z'}$, i.e. all the H_i, are suppressed the set of fields that may obtain VEVs is divided into two classes:

1. Singlets from the NS sector and the sector $S+b_1+b_2+\alpha+\beta$. These states transform solely under the observable gauge group and are $SO(10) \times SO(16)$ singlets.

2. States from the sectors $b_j + 2\gamma$, $j = 1, 2, 3$. These states are obtained from the twisted sectors of the orbifold model. They are $SO(10)$ singlets and transform under the hidden gauge group as the 16 representation of $SO(16)$ decomposed under $SU(5) \times SU(3) \times U(1)^2$.

If we restrict the allowed VEVs only to states from the first class, then the cubic level F–flat solution is preserved to all orders of nonrenormalizable terms. This result follows from the charges of the states from these sectors under the left–moving global $U(1)_{\ell_{1,2,3}}$ symmetries. The order N terms that have to be investigated are of the form,

$$\langle(\alpha\beta)^j(NS)^{N-j}\rangle \qquad\qquad (j = 4, \cdots, N), \tag{12}$$

where (NS) denotes fields from the Neveu–Schwarz sector and $(\alpha\beta)$ denotes fields that belong to the sector $b_1 + b_2 + \alpha + \beta$. Without loss of generality, we can choose two of the $(\alpha\beta)$ fields to be the two space–time fermions in these correlators. The $U(1)_{\ell_{1,2,3}}$ charges for the $(\alpha\beta)$ fields are $(0, 0, \frac{1}{2})$ for the fermions and $(-\frac{1}{2}, -\frac{1}{2}, 0)$ for the bosons. All NS fields in Eq. (12) are bosonic fields with charges $U(1)_{\ell_j} = 0$ or -1. Of the NS singlets, only Φ_{12}, $\bar{\Phi}_{12}$ and ξ_3 carry $U(1)_{\ell_3}$ charges. We can always choose a basis in which the $U(1)_{\ell_3}$ charge of these fields is picture changed to zero. The picture changing operation on the $(\alpha\beta)$ scalars can only change them to $(\pm\frac{1}{2}, \pm\frac{1}{2}, 0)$. Therefore, none of the terms of the form of Eq. (12) are invariant under $U(1)_{\ell_3}$. The conclusion is that all these terms vanish identically to all orders. Thus, if we allow only VEVs for the states from the NS sector and the sector $b_1+b_2+\alpha+\beta$, there are no new F terms even if we include nonrenormalizable terms to all orders. Consequently, in this case, the cubic level F and D–flat solution is valid to all orders N and supersymmetry is unbroken.

However, to obtain a Cabibbo angle of the correct order of magnitude it is necessary to give VEVs to some states from the sector $b_j + 2\gamma$. This result follows from the following considerations. The Higgs doublets h_3 and $\bar{h}_3$ obtain a Planck scale mass due to the cubic level F and D–flatness constraints [10,12]. As a result, and because the horizontal charges forbid the states from the sector b_3 to couple

directly to the remaining Higgs doublets, the states from the sector b_3 are identified with the lightest generation, while the sectors b_1 and b_2 are identified with the two heavier generations. However, due to the $U(1)_{\ell_{j+3}}$ horizontal symmetries, mixing terms between the sectors b_1, b_2 and b_3 are only obtained by exchanging states from the sector $b_j + 2\gamma$ [12]. Consequently, to obtain a Cabibbo angle that is not too small necessitates that we give VEVs of order $M/10$ to some states from the sector $b_j + 2\gamma$ [14]. Thus, when analyzing the effective supergravity theory that is obtained from these models, we have to include these hidden sector VEVs as well. In the next section, we show that the same VEVs must be imposed to obtain a nonsingular mass matrix for the hidden $SU(5)$ matter states which is essential for a stable supersymmetric vacuum [25]. Thus, the same VEVs that generate the Cabibbo mixing also guarantee that the supersymmetric vacuum is well defined.

We therefore have to find F and D flat solutions that contain nonvanishing VEVs for the states from the sectors $b_j + 2\gamma$. In the following, to illustrate our scenario, we consider a specific F and D flat solution, which corresponds to a specific choice of string vacuum at M_{Pl}. We do not know the string dynamics that select the string vacuum but simply consider one. An explicit solution that satisfies all the cubic level F and D flatness constraints is given by the following set of nonvanishing VEVs,

$$\{\bar{V}_2, V_3, \Phi_{45}, \bar{\Phi}_{13}, \bar{\Phi}_1^-, \Phi_2^+, \bar{\Phi}_3^-, \xi_1, \xi_2\}, \tag{13}$$

with

$$2|\langle\bar{V}_2\rangle|^2 = 2|\langle V_3\rangle|^2 = \frac{1}{4}|\langle\Phi_{45}\rangle|^2 = |\langle\bar{\Phi}_{13}\rangle|^2 = \frac{g^2}{16\pi^2}\frac{1}{2\alpha'}, \tag{14a}$$

$$2|\langle\bar{\Phi}_1^-\rangle|^2 = 2|\langle\Phi_2^+\rangle|^2 = |\langle\bar{\Phi}_3^-\rangle|^2 = \frac{g^2}{16\pi^2}\frac{1}{2\alpha'}, \tag{14b}$$

The VEVs of ξ_1 and ξ_2 are not fixed by the F and D constraints. In general $\langle\xi_1, \xi_2\rangle = \mathcal{O}(g^2 M/4\pi)$ must be imposed to obtain realistic quark and lepton masses [10,12]. The fields $\{\Phi_{12}, \bar{\Phi}_{12}, \Phi_{13}, \bar{\Phi}_{13}, \Phi_{23}, \bar{\Phi}_{23}\}$ are related to the untwisted moduli fields of the $Z_2 \times Z_2$ orbifold. Their VEVs in the fermionic model are determined

by the F and D–flatness requirements. From Eq. (14a) and with $|\langle\bar{\Phi}_{23}\rangle|^2 = |\langle\hat{\eta}_1\rangle|^2$, $|\langle\bar{\Phi}_{13}\rangle|^2 = |\langle\hat{\eta}_2\rangle|^2$, and $|\langle\bar{\Phi}_{12}\rangle|^2 = |\langle\hat{\eta}_3\rangle|^2$, it is seen that we have $|\langle\hat{\eta}_j\rangle|^2 = \{0, g^2/(16\pi^2), 0\}$, in units of M. Inserting these values into Eq. (2) for the Kähler function, we observe that $\langle\eta_j^0\rangle \approx \sqrt{2},\ j = 1,2,3$. Hence, the analogue of the conventional moduli fields are determined to be of order one.

The hidden sector of the superstring model contains two non–Abelian gauge groups, $SU(3)$ and $SU(5)$. The $SU(3)$ group is broken by our choice of VEVs and does not play a role in our analysis of supersymmetry breaking. The massless spectrum contains four pairs of $5+\bar{5}$ of the hidden $SU(5)$ gauge group. Of those H_{19} and H_{20} obtain Planck scale masses due to the cubic level term $H_{19}H_{20}\xi_2$. Thus we obtain below the Planck scale a $SU(5)$ gauge group with three pairs of $5+\bar{5}$.

4. Supersymmetry breaking from nonrenormalizable terms

In the previous section we showed that the solution to the cubic level F and D constraints divide into two classes: those that include states from the sectors $b_j+2\gamma$ and those that do not. For the second class of solutions, there are no corrections from nonrenormalizable terms to the cubic level F and D–flat solution. However, the second class of solutions cannot produce realistic quark mixing. Therefore, the realistic solution can only be of the first class. Furthermore, the unbroken non–Abelian hidden sector gauge groups (which in our case is hidden $SU(5)$) condense at some scale Λ_H. Then hidden matter condensates foenormalizable terms in the superpotential and taking into account the nonperturbative effects of the hidden $SU(5)$ gauge group. We find that contrary to the second class solutions, in this case the nonrenormalizable terms modify the cubic level constraints. We investigate this modification and argue that when nonrenormalizable terms with hidden sector condensates are taken into account the set of F constraints cannot be satisfied simultaneously. As a result nonvanishing F–terms are generated and supersymmetry is broken.

We search for allowed nonrenormalizable terms that contain $SO(10)$ singlet fields from the NS, $\alpha\beta$ and $b_j+2\gamma$ sectors. At order $N=5$ we find,

$$V_2\bar{V}_2\Phi_{45}\Phi_2^-\xi_1, \qquad V_1\bar{V}_1\Phi_{45}\bar{\Phi}_1^+\xi_2, \tag{15a,b}$$
$$T_2\bar{T}_2\Phi_{45}\Phi_2^+\xi_1, \qquad T_1\bar{T}_1\Phi_{45}\bar{\Phi}_1^-\xi_2, \tag{15c,d}$$

at order N=7,

$$T_2\bar{T}_3V_3\bar{V}_2\Phi_{45}\Phi_{45}\bar{\Phi}_{13}, \tag{16a}$$
$$T_1\bar{T}_2V_1\bar{V}_2\Phi_{45}\Phi_{45}\xi_1, \tag{16b}$$
$$T_2\bar{T}_1V_1\bar{V}_2\Phi_{45}\Phi_{45}\xi_2, \tag{16c}$$

$$V_2\bar{V}_2\Phi_{45}\Phi_2^-\xi_1[(\frac{\partial W_3}{\partial\xi_3})+\xi_i\xi_i+\Phi_{13}\bar{\Phi}_{13}+\Phi_{23}\bar{\Phi}_{23})], \tag{16d}$$
$$V_1\bar{V}_1\Phi_{45}\bar{\Phi}_1^+\xi_2[(\frac{\partial W_3}{\partial\xi_3})+\xi_i\xi_i+\Phi_{13}\bar{\Phi}_{13}+\Phi_{23}\bar{\Phi}_{23})], \tag{16e}$$
$$V_2\bar{V}_2\Phi_{45}\bar{\Phi}_2^+\xi_1(\frac{\partial W_3}{\partial\Phi_{12}}), \tag{16f}$$
$$V_1\bar{V}_1\Phi_{45}\Phi_1^-\xi_2(\frac{\partial W_3}{\partial\bar{\Phi}_{12}}), \tag{16g}$$

and at order N=8,

$$\bar{T}_2T_3V_3\bar{V}_2\Phi_{45}\Phi_{45}\bar{\Phi}_{13}\xi_1, \tag{17a}$$
$$T_1\bar{T}_3V_1\bar{V}_3\Phi_{45}\Phi_{45}\bar{\Phi}_{23}\xi_2, \tag{17b}$$
$$T_2\bar{T}_3V_2\bar{V}_3\Phi_{45}\Phi_{45}\bar{\Phi}_{13}\xi_1, \tag{17c}$$
$$T_3\bar{T}_1V_3\bar{V}_1\Phi_{45}\Phi_{45}\bar{\Phi}_{23}\xi_2. \tag{17d}$$

First, we examine the terms that include in addition to NS and $\alpha\beta$ states also $V_{1,2,3}$ and $\bar{V}_{1,2,3}$. To produce realistic quark mixing some V_j and $\bar{V}_j$ must obtain VEVs in the cancelation of the "anomalous" $U(1)$ D–term equation. Eqs. (15a,b)

indicate that giving VEVs to both V_j and $\bar{V}_j$ from a sector $b_j + 2\gamma$ may result in too large a supersymmetry breaking scale in the observable sector. Therefore, we require that the order $N = 5$ terms, Eqs. (15a,b), vanish as well as all of their derivatives. This imposes additional constraints on the allowed VEVs, namely, $\langle \Phi_2^- \rangle = 0$ and $\langle \bar{\Phi}_1^+ \rangle = 0$ and either V_j or $\bar{V}_j$ from each sector $b_j + 2\gamma$ can get a VEV. Then, the $N = 5$ and $N = 7$ order terms, of the form $V_j \bar{V}_j \phi^n$, as well as all of their derivatives, vanish due to our choice of flat directions and the cubic level constraints $(\partial W_3/\partial \phi_i) = 0$. We then find that there are no additional nontrivial constraints from nonrenormalizable terms up to order $N = 11$.

Next we examine the terms that contain 5 and $\bar{5}$ of the hidden $SU(5)$ gauge group, *i.e.* $T_{1,2,3}$ and $\bar{T}_{1,2,3}$. The scale at which the hidden $SU(5)$ gauge group becomes strongly interacting is given by

$$\Lambda_5 = M \exp(\frac{2\pi}{b} \frac{(1-\alpha_0)}{\alpha_0}), \tag{18}$$

where $b = \frac{1}{2} n_5 - 15$. For $n_5 = 6$ and $\alpha_0 = (1/25 - 1/20)$, $\Lambda_5 \sim (10^{12} - 10^{14}) GeV$. The value of α_0 assumes that the dilaton potential has a minimum with $\langle S \rangle \sim 1/2$. The T's and $\bar{T}$'s will form hidden sector condensates when the hidden $SU(5)$ gauge group becomes strongly interacting. To evaluate the gaugino and matter condensates we use the well known expressions for supersymmetric $SU(N)$ with matter in $N + \bar{N}$ representations [25],

$$\frac{1}{32\pi^2} \langle \lambda\lambda \rangle = \Lambda^3 \left(det \frac{\mathcal{M}}{\Lambda} \right)^{1/N}, \tag{19a}$$

$$\Pi_{ij} = \left\langle \bar{T}_i T_j \right\rangle = \frac{1}{32\pi^2} \langle \lambda\lambda \rangle \, \mathcal{M}_{ij}{}^{-1}, \tag{19b}$$

where $\langle \lambda\lambda \rangle$, $\mathcal{M}$ and Λ are the hidden gaugino condensate, the hidden matter mass matrix and the $SU(5)$ condensation scale, respectively. Modular invariant generalization of Eqs. (19a,b) for the string case were derived in Ref. [26]. The nonrenormalizable terms can be put in modular invariant form by following the

procedure outlined in Ref. [28]. Approximating the Dedekind η function by $\eta(\hat{T}) \approx e^{-\pi\hat{T}/12}(1-e^{-2\pi\hat{T}})$ we verified that the calculation using the modular invariant expression from Ref. [26] (with $\langle\hat{T}\rangle \approx M$) differ from the results using Eq. (19), by at most an order of magnitude. Therefore, for the purpose of our qualitative observations the use of Eqs. (19a,b) is adequate.

In our case the matrix $\mathcal{M}$ is given by,

$$\mathcal{M} = \begin{pmatrix} 0 & C_1 & 0 \\ B_1 & A_2 & C_2 \\ 0 & C_3 & A_1 \end{pmatrix} \, , \tag{20}$$

where A, B, C arise from terms at orders $N = 5, 8, 7$ respectively and are given by

$$A_1 = \frac{\langle \Phi_{45}\bar{\Phi}_1^{-}\xi_2\rangle}{M^2}, \qquad A_2 = \frac{\langle \Phi_{45}\Phi_2^{+}\xi_1\rangle}{M^2}, \tag{21a,b}$$

$$B_1 = \frac{\langle V_3\bar{V}_2\Phi_{45}\Phi_{45}\bar{\Phi}_{13}\xi_1\rangle}{M^5}, \tag{21c}$$

$$C_1 = \frac{\langle V_3\bar{V}_2\Phi_{45}\Phi_{45}\bar{\Phi}_{13}\rangle}{M^4}, \quad C_2 = \frac{\langle V_1\bar{V}_2\Phi_{45}\Phi_{45}\xi_1\rangle}{M^4}, \tag{21d,e}$$

$$C_3 = \frac{\langle V_1\bar{V}_2\Phi_{45}\Phi_{45}\xi_2\rangle}{M^4} \quad . \tag{21f}$$

For the solution given by Eq. (14) $V_1 = 0 \Rightarrow C_2 = C_3 = 0$. Taking generically $\langle\phi\rangle \sim gM/4\pi \sim M/10$ we obtain $A_i \sim 10^{15}\ GeV$, $B_i \sim 10^{12}\ GeV$, and $C_i \sim 10^{13}\ GeV$. The matrix $\mathcal{M}^{-1}$ is given by,

$$\mathcal{M}^{-1} = \begin{pmatrix} -\frac{A_1A_2 - C_1C_2}{A_1B_1C_1} & \frac{1}{B_1} & -\frac{C_2}{A_1B_1} \\ \frac{1}{C_1} & 0 & 0 \\ -\frac{C_3}{A_1C_1} & 0 & \frac{1}{A_1} \end{pmatrix} \, . \tag{22}$$

The zeros in $\mathcal{M}^{-1}$ indicate that of the above nonrenormalizable terms only the ones with $T_1\bar{T}_1$ and $T_2\bar{T}_3$ and $\bar{T}_2T_3$ remain and the rest vanish. For the specific solution that we consider Π_{13} and Π_{31} vanish as well. This does not affect our results because these condensates do not appear in the nonrenormalizable terms

at least up to order $N = 8$. To obtain a stable vacuum the hidden matter mass matrix must be nonsingular, *i.e.* the determinant of $\mathcal{M}$ must not vanish [25]. To obtain a nonsingular determinant we must impose some nonvanishing VEVs. From Eqs. (20) and (21) we find that in particular we must impose that $\bar{V}_2$ and V_3, as well as $\{\Phi_{45}, \Phi_2^+, \bar{\Phi}_1^-, \bar{\Phi}_{13}, \xi_{1,2}\}$, get nonvanishing VEVs. We observe that a stable supersymmetric vacuum requires hidden sector VEVs. It is interesting to note that the same constraint that must be imposed to obtain realistic quark mixing has to be imposed to obtain a stable supersymmetric vacuum. With the cubic level solution given by Eq. (14), $\mathcal{M}$ is nonsingular and its determinant is equal to $\text{Det}\mathcal{M} = -A_1 B_1 C_1$.

We are now ready to estimate the scale of supersymmetry breaking. Neglecting the effects of the electroweak symmetry breaking, we have

$$V_{eff} = e^G (G_i^j)^{-1} G_j G^i - 3e^G, \tag{23}$$

where $G = K + ln|W|^2$ and K and W are given by Eqs. (6) and (1, 12–14) respectively. Here the subscript (superscript) i denotes $\partial/\partial\phi_i$ ($\partial/\partial\phi_i^\dagger$). Then,

$$V_{eff} = (G_i^i)^{-1} |F_i|^2 - 3e^K |W|^2, \tag{24}$$

where

$$F_i = e^{K/2}(W_i + WK_i), \tag{25}$$

In general, inclusion of the nonrenormalizable terms in the superpotential and condensation in the hidden sector modifies the effective cubic superpotential. As a result, there is a new vacuum corresponding to the new effective potential. The old F and D flat solution given by Eq. (14) will no longer correspond to a supersymmetric minimum. In fact, now the solution given by Eq. (14) is neither supersymmetric nor a minimum of the new effective potential. To demonstrate explicitly that supersymmetry is broken, we would have to write down the effective

superpotential for all the fields in the massless spectrum, minimize the effective potential and show that in the vacuum one of the F–terms, Eq. (25), is different from zero. In this case supersymmetry is spontaneously broken in the vacuum of V_{eff}. However, due to the large number of fields and the complicated superpotential, this is an intractable task to perform analytically. A numerical analysis has to be employed and is deferred to future work. Instead, we resort to the following argument to show that supersymmetry is broken in the new vacuum. The nonrenormalizable terms modify the cubic level superpotential. We first focus on the modification due to the quintic order terms. From Eq. (15) and Eq. (22) we observe that at the quintic order there is a single term that modifies the cubic level superpotential, $W_5 = c\Pi_{11}\Phi_{45}\bar{\Phi}_1^-\xi_2$. The coefficient c is expected to be of order one [19]. This term modifies the cubic level superpotential and the equations for $\partial W/\partial\Phi_{45}$, $\partial W/\partial\bar{\Phi}_1^-$, $\partial W/\partial\xi_2$ where now $W = W_3 + W_5$.

We see that $\langle W_5\rangle \neq 0$ for the explicit F and D flat direction given by Eq. (14). (We remind that with Eq. (14) $\langle W_3\rangle = 0$). $\langle W_5\rangle$ is necessarily nonvanishing for the following reasons. It is always possible to rewrite the D–term equations in a way that the only nonvanishing coefficient in the equation for D_A is the coefficient of $|\langle\Phi_{45}\rangle|^2$. Thus, $\langle\Phi_{45}\rangle$ must be different from zero. In addition, if $\langle\bar{\Phi}_1^-\rangle$ and $\langle\xi_2\rangle$ vanish the hidden matter mass matrix is singular and the vacuum exhibits a runaway behavior [25]. If we insist that the determinant of the matter condensates is nonvanishing so that the vacuum is well defined, we find that $\langle W_3 + W_5\rangle \neq 0$.

The modified F equations for $W = W_3 + W_5$ read,

$$\frac{\partial W}{\partial\bar{\Phi}_1^-} = \bar{\Phi}_1^+\bar{\Phi}_{12} + \frac{1}{2}\xi_3\Phi_1^- + c\Pi_{11}\Phi_{45}\xi_2 + c\Phi_{45}\bar{\Phi}_1^-\xi_2\frac{\partial\Pi_{11}}{\partial\bar{\Phi}_1^-} = 0 \ , \qquad (26a)$$

$$\frac{\partial W}{\partial\Phi_{45}} = \frac{1}{2}\xi_3\bar{\Phi}_{45} + c\Pi_{11}\bar{\Phi}_1^-\xi_2 + c\Phi_{45}\bar{\Phi}_1^-\xi_2\frac{\partial\Pi_{11}}{\partial\Phi_{45}} = 0 \ , \qquad (26b)$$

$$\frac{\partial W}{\partial\xi_2} = H_{13}H_{14} + H_{17}H_{18} + c\Pi_{11}\bar{\Phi}_1^-\Phi_{45} + c\Phi_{45}\bar{\Phi}_1^-\xi_2\frac{\partial\Pi_{11}}{\partial\xi_2} = 0 \ . \qquad (26c)$$

The first two terms in these equations arise from the cubic level superpotential whereas the last two give the corrections from W_5. The last term in each equation

arises due to the implicit field dependence of the matter condensate Π_{11}. Π_{11} depends on the fields in two ways as can be seen from Eq. (19b): through the gaugino condensate $\langle\lambda\lambda\rangle$ which depends on $det\mathcal{M}$ and through $\mathcal{M}^{-1}_{ij}$ given by Eq. (22). The other cubic level constraints given by Eqs. (8) and (10) remain intact. An explicit calculation of the last term in each equation shows that the corrections to the F equations due to W_5 are given by

$$\frac{\partial W_5}{\partial \bar{\Phi}_1^-} = \frac{1}{5}\Pi_{11}\Phi_{45}\xi_2, \tag{27a}$$

$$\frac{\partial W_5}{\partial \Phi_{45}} = \Pi_{11}\bar{\Phi}_1^-\xi_2, \tag{27b}$$

$$\frac{\partial W_5}{\partial \xi_2} = \frac{1}{5}\Pi_{11}\Phi_{45}\bar{\Phi}_1^-. \tag{27c}$$

We see that the last two terms in each equation are nonzero. The other cubic level equation remain intact. The cubic level constraint Eq. (8g) and Eq. (8d,e) still require $\langle\Phi_{12}\rangle = \langle\bar{\Phi}_{12}\rangle = \langle\xi_3\rangle = 0$. But then the equations obtained from the quintic order modification cannot be satisfied, unless $\langle\Phi_{45}\rangle = \langle\bar{\Phi}_1^-\rangle = \langle\xi_2\rangle = 0$. However, it is always possible to rewrite the D–term equations in a way that the only nonvanishing coefficient in the equation for D_A is the coefficient of $|\langle\Phi_{45}\rangle|^2$. Thus, $\langle\Phi_{45}\rangle$ must be different from zero. In addition, if $\langle\bar{\Phi}_1^-\rangle$ and $\langle\xi_2\rangle$ vanish the hidden matter mass matrix is singular and the vacuum exhibits a runaway behavior [25]. If we insist that the vacuum is well defined and that the determinant of the matter condensates is nonvanishing, we see that there is no set of VEVs that satisfies the new set of F constraints, $\partial(W_3+W_5)/\partial\phi_i = 0$, up to $N=5$. Consequently, for all possible choices of VEVs, $\partial W/\partial\phi_i \neq 0$, for some ϕ_i. As long as V_{eff} has a minimum, $\partial W/\partial\phi_i \neq 0$ in the minimum.

The above arguments apply to all orders $N>3$. In fact, we can demonstrate this by considering the $N=7$ term, W_7, and its corrections to the cubic level and $N=5$ F constraints. An analysis similar to the above for the $N=5$ case shows that $\langle W_7\rangle \neq 0$ if we require a stable vacuum. In addition, the F equations for $\bar{V}_2, V_3, \Phi_{45}, \bar{\Phi}_{13}, \xi_1$ are modified due to W_7 because of the dependence of Π_{32}

on these fields through the gaugino condensate and $\mathcal{M}^{-1}$. One can show that now, these modified equations cannot be satisfied simultaneously with the ones left unchanged. We conclude that $\langle W\rangle \neq 0$ and $\langle W_i\rangle \neq 0$ at order $N=7$ too. Therefore, we expect that, generically $\langle W\rangle \neq 0$ and $\langle W_i\rangle \neq 0$ (for some fields ϕ_i) at some order $N>3$.

To show that supersymmetry is broken, we need to show that, at a given order N, $\langle F_i\rangle = exp(\langle K\rangle/2)\langle (WK_i+W_i)\rangle \neq 0$. We will show that $\langle F_i\rangle$ obtained from the modified superpotential, $W=W_3+\ldots+W_N$, is dominated by the $\langle W_i\rangle$ piece. (This unless $\langle W\rangle$ arises from a lower order than $\langle W_i\rangle$. We will consider this case which is not the one at hand separately later.) Then, whether $\langle W\rangle$ vanishes or not at that order, the first term in $\langle F_i\rangle$ cannot cancel the second one. In our case, $\langle W_i\rangle \neq 0$ for $i=\Phi_{45},\bar{\Phi}_1^-,\xi_2$ up to order $N=5$. We expect that at higher orders there will be other fields ϕ_i with $\langle W_i\rangle \neq 0$ as a result of nonrenormalizable terms in the superpotential. (For example the $N=7$ terms give in addition to the above fields $i=\bar{V}_2,V_3,\bar{\Phi}_{13}$.) Assuming $\langle \phi_i\rangle \approx M/10$, for matter with $K=-c\langle \phi_i\phi_i^\dagger\rangle/M^2$ and c of order one

$$\langle exp(K/2)\rangle \approx exp(\frac{-\langle \phi_i\phi_i^\dagger\rangle}{M^2}) \approx exp(-\frac{1}{200}) \sim 1, \tag{28}$$

where we took the dilaton and moduli VEVs to be of order M. Writing the powers of M explicitly,

$$\langle WK_i\rangle \sim \langle W_i\rangle \frac{\langle \phi_i\phi_i^\dagger\rangle}{M^2} \approx 10^{-2}\langle W_i\rangle. \tag{29}$$

So the first term in $\langle F_i\rangle$ is suppressed by a factor of $\langle \phi_i\phi_i^\dagger\rangle/M^2 \approx 10^{-2}$ with respect to the second term at each order separately. The first term in $\langle F_i\rangle$ can cancel the second one either if it contains fields with VEVs larger than M_{Pl} or if it arises from a lower order. We disregard the first possibility because in this case the truncation of nonrenormalizable terms at any order is inadequate. From the relative magnitude of the two terms (~ 100) one sees that if $\langle W\rangle$ arises from order $N-2$ where $\langle W_i\rangle$ arises from order N the two terms in $\langle F_i\rangle$ will be of the

same magnitude and they might cancel each other. This does not happen above, since in our case both $\langle W\rangle$ and $\langle W_i\rangle$ become nonzero at the same order, $N=5$. In general, such a cancellation requires a high degree of fine tuning of the scalar VEVs and may even be excluded due to the other F and D constraints.

In our case, both $\langle W\rangle$ and $\langle W_i\rangle$ become nonzero at $N=5$ and the W_i piece (from $N=5$) dominates $\langle F_i\rangle$. Consequently, $\langle F_i\rangle \sim \langle W_i\rangle \Rightarrow \langle F_i\rangle \neq 0$ if $\langle W_i\rangle \neq 0$ and supersymmetry is broken by the ϕ_i (matter) F term. We have argued above that $\langle F_i\rangle \neq 0$ for all sets of VEVs or minima of V_{eff}. This means that supersymmetry cannot be preserved by any of the vacua of the new, modified superpotential. As long as V_{eff} has a minimum, supersymmetry will be broken in that minimum. Although we cannot show its existence explicitly, we assume that V_{eff} has at least one minimum (which is the new vacuum).

For simplicity we considered the corrections to W_3 only up to $N=5$. However, our arguments are valid to all orders N. In fact, if for some cubic level flat direction the nonrenormalizable terms up to order $N=5$ vanish, the same mechanism will break supersymmetry at some higher order (e.g. $N=7$ in our case). We expect that once supersymmetry is broken at some order N, terms from higher orders cannot restore it. The reason is that higher order terms need to include scalars with VEVs larger than the Planck scale M_{Pl}. For example, since $N=7$ order terms are suppressed by 10^{-2} with respect to $N=5$ order terms, to cancel $\langle (W_3+W_5)_i\rangle$ from $N=7$ terms we need, for some scalars which appear in the $N=7$ order terms, $\langle\phi\rangle \sim 10^2(M/10) \sim 10M \sim M_{Pl}$. However, generically the VEVs are of order $g^2M/4\pi \sim M/10$. The situation is complicated in the case with matter condensates because of the Π_{ij} dependence on $\mathcal{M}^{-1}_{ij}$. However, even if, due to the matter condensates, some term happens to be of the same magnitude as some lower order term, without a high degree of fine tuning, we again expect that if supersymmetry is broken at some order N, it is not restored by higher orders.

To estimate the magnitude of supersymmetry breaking in the observable sector,

we estimate the gravitino mass which is given by

$$m_{3/2} = \frac{1}{M^2} e^{\langle K/2 \rangle} |\langle W \rangle|, \tag{31}$$

With $\Lambda_5 \sim 10^{13}$ GeV the gaugino condensate is estimated from Eq. (19a) to be $\sim 10^{39}$ GeV^3. For the matter condensates we get from Eq. (19b) and Eq. (22),

$$\Pi_{11} \approx 10^{24}\ GeV^2 \qquad \Pi_{23} \approx 10^{26}\ GeV^2 \qquad \Pi_{32} \approx 10^{27}\ GeV^2 \tag{32}$$

Taking $\langle \phi \rangle / M \sim 1/10$ we get $m_{3/2} \sim 1$ TeV from the $N = 5$, $m_{3/2} \sim 10$ TeV from $N = 7$ order terms and $m_{3/2} \sim 100$ GeV from the $N = 8$ order terms. Using the modular invariant expressions for the gauge and matter condensates from Ref. [26] and multiplying the nonrenormalizable terms by appropriate factors of the Dedekind η function, as outlined in Ref. [19], yields results that differ from ours at most by an order of magnitude which is tolerable for our qualitative estimates.

5. Discussion and conclusions

In this paper, we examined the problem of supersymmetry breaking in a class of realistic superstring derived standard–like models. We presented a possible scenario for supersymmetry breaking that takes into account modifications of the cubic level superpotential due to nonrenormalizable terms and hidden sector matter condensates. Hidden matter condensates form when the hidden gauge group becomes strong at a scale, $\Lambda_H \sim 10^{12-14}\ GeV$. At the same scale, gaugino condensates form as well. Supersymmetry breaking due to gaugino condensation has been extensively studied in the literature [33]. Is it possible to say under what conditions either of the two supersymmetry breaking mechanisms is dominant? Supersymmetry is broken when

$$\langle F_i \rangle = \langle e^G (G_i^j)^{-1} G_j \rangle + \langle f_{\alpha\beta,i} \lambda^\alpha \lambda^\beta \rangle \neq 0 \ . \tag{33}$$

At the string tree level $f_{\alpha\beta} = S\delta_{\alpha\beta}$ and the second term is nonzero only for $i = S$. We want to estimate the relative magnitudes of these two contributions. The first and the second terms in $\langle F_i \rangle$ arise from, scalar matter condensation and from gaugino condensation, respectively. The values of the gaugino and matter condensates are given by Eqs. (19a,b). Note that Π_{ij} and therefore F_i from matter condensation are proportional to the gaugino condensate $\langle \lambda^\alpha \lambda^\beta \rangle$. Taking generically, $\langle \phi \rangle \sim M/10$ and using the fact that the mass matrix $\mathcal{M}$ arises from order $N > 3$ terms, we get from nonrenormalizable terms of order L,

$$W_L = \Pi_{ij} \frac{\langle \phi \rangle^{L-2}}{M^{L-3}} \quad \text{and} \quad \Pi_{ij} \sim \frac{\langle \lambda^\alpha \lambda^\beta \rangle}{32\pi^2} \frac{M^{N-3}}{\langle \phi \rangle^{N-2}} \ . \tag{34}$$

Therefore,

$$\langle F_\phi \rangle \sim \frac{\langle \lambda^\alpha \lambda^\beta \rangle}{32\pi^2} \left(\frac{\langle \phi \rangle}{M} \right)^{L-N} \frac{1}{\langle \phi \rangle} \ . \tag{35}$$

The contribution of the gaugino condensate to the F–term is simply given by $\langle F_g \rangle \sim \langle \lambda^\alpha \lambda^\beta \rangle / 32\pi^2 M$. We expect, by comparing the two expressions, that matter condensation effects are dominant when $L - N < 1$ and vice versa. In general,

whether one or the other supersymmetry breaking mechanisms is dominant depends not only on the hidden gauge group that condenses and its matter content but also on the orders at which hidden matter gets mass and F equations are violated as given by the relation $L - N < 1$. In this respect our model is on the borderline, i.e. the contributions of hidden matter and gaugino condensation to supersymmetry breaking are comparable since in our case $L \sim N$.

A fundamental problem in all supersymmetry breaking mechanisms is the stability of the dilaton vacuum. Perturbative coupling unification requires that $g^2(M_{Pl}) \sim 1/4\langle S\rangle \sim 1/2$. We have not calculated V_{eff} explicitly and therefore cannot say whether or not in our case the dilaton potential is stable with the required minimum. It has been previously noted that one can stabilize the dilaton potential if there are two hidden gauge groups which condense. The superstring standard–like models typically admit two non–Abelian hidden gauge groups, $SU(5) \times SU(3)$. Of those one has to be broken at M by VEVs that are needed to obtain realistic quark mixing and a stable vacuum. In the scenario that we considered the hidden $SU(3)$ is broken by these VEVs and therefore dilaton stabilization by two (or more) non–Abelian hidden gauge groups cannot be applied. However, there exists a possibility to break the hidden $SU(5)$ gauge group, for example to $SU(3) \times SU(2) \times U(1)$ or to $SU(4) \times U(1)$, in which case it may be possible to obtain two hidden gauge groups with matter content and similar beta functions. In this case we may have a problem with vacua for which the number of, $N \oplus \bar{N}$ flavor pairs exceeds, N, the number of colors. Recent progress on nonperturbative supersymmetric field theories may be instrumental in this case [31]. Another possibility is to try to build the effective superpotential in terms of the dilaton, S and the moduli, $\hat{T}_i$. For example, the nonrenormalizable terms produce a moduli dependence from the Dedekind η functions which are included to get modular weight -3. The dilaton dependence arises from $\Lambda \sim exp(-8\pi^2 S/b)$, which appears in Π_{ij}, and from $\langle\phi\rangle \sim g^2 M^2/4\pi \sim M^2/4\pi S$. One can try to build these effective terms and minimize them for S and $\hat{T}$.

Another fundamental problem is the value of the cosmological constant (Λ_C)

after supersymmetry breaking. The condition for a vanishing cosmological constant is, from Eq. (23),

$$\langle (G_i^j)^{-1} G_j G^i \rangle = 3 \ , \tag{36}$$

where a summation over i, j is implied. This is an additional constraint that the supersymmetry breaking vacuum must satisfy. The problem with this constraint is that an approximate (order by order in N) calculation is meaningless since a negligible effect of order m/M (where $m << M$) results in $\Lambda_C \sim m^2 M^2$ which is still huge. We expect that in our model $\Lambda_C \neq 0$ and we do not have much to add on this point.

A novel feature of free fermionic models is the existence of horizontal gauge $U(1)_r$ symmetries under which both the observable and hidden matter states which eventually condense are charged. Therefore, supersymmetry breaking in the hidden sector can be communicated to the observable sector not only by gravity (which is universal) but also by different $U(1)_r$ which may couple the observable and hidden sectors nonuniversally. This coupling will depend on the $U(1)_r$ charges of the relevant states and the mass of the $U(1)_r$ gauge boson. Both of these can be slightly different for different observable matter states. Thus, in these models one may obtain different soft supersymmetry breaking masses for the scalars.

The low–energy phenomenology which arises from supersymmetry breaking due to nonrenormalizable terms and hidden matter condensation is particularly interesting. In general, the gaugino and matter condensates generate nonvanishing F–terms for the dilaton, the moduli and the matter fields. The various F–terms produce different forms of soft supersymmetry breaking terms. The dominant supersymmetry breaking F–term will determine the supersymmetric mass spectrum and consequently the low energy phenomenology. For example, if the dilaton F–term dominates, squark masses are universal [34], while if the moduli terms dominate, in general, one expects the squark masses to be nonuniversal [35]. For specific cubic level F and D flat solutions, like Eq. (14), it may be possible, as

we argued above, to determine the dominant term and therefore to make specific predictions for the supersymmetric mass spectrum.

In this paper we examined the problem supersymmetry breaking in the superstring derived standard–like models. The problem of supersymmetry breaking has two different aspects. The first is the determination of the compactification parameters, i.e. the dilaton, the moduli and the remaining $SO(10)$ singlet VEVs. The determination of this parameters must await a better understanding of superstring theory both at the perturbative as well as the nonperturbative level. It may be futile, in our opinion, to try to determine these parameters only by incorporating nonperturbative effects in the effective point field theory. However, if we accept this deficiency, while seeking a better understanding of string theory we can parameterize our ignorance into several parameters like the dilaton VEV, the moduli VEVs and the singlet VEVs. We showed that with these parameters fixed supersymmetry breaking arises due to the existence of a non–Abelian hidden gauge group with matter in vector–like representations. The modification of the superpotential due to the strong hidden sector dynamics and nonrenormalizable terms may result in hierarchical supersymmetry breaking in the observable sector. The resulting soft supersymmetry breaking terms can then be obtained and specific predictions for the supersymmetric spectrum be made. The existence of supersymmetric particles near the TeV scale will be decided by future experiments. In the event that supersymmetric particles are observed the sparticle spectrum will be used to restrict the SUSY breaking parameters and hence will be used to study the compactification parameters and their determination by possibly nonperturbative string effects. We will return to the pursuit of these ideas in future publications.

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